Transformations in 2-D

- Translation → Change in position
- Scaling → Change in size, shape
- Rotation → Movement along circular path
- Reflection → सबको पता होगा
- Shearing → Ye aage btaunga

Transformation : Transformation means changes in orientation, size & shape of the object. They are used to position the object, to change the shape of object and even to change how something is viewed.

The basic geometrical transformations are :

① Traslation
② Rotation
③ Scaling

& other derived transformations are

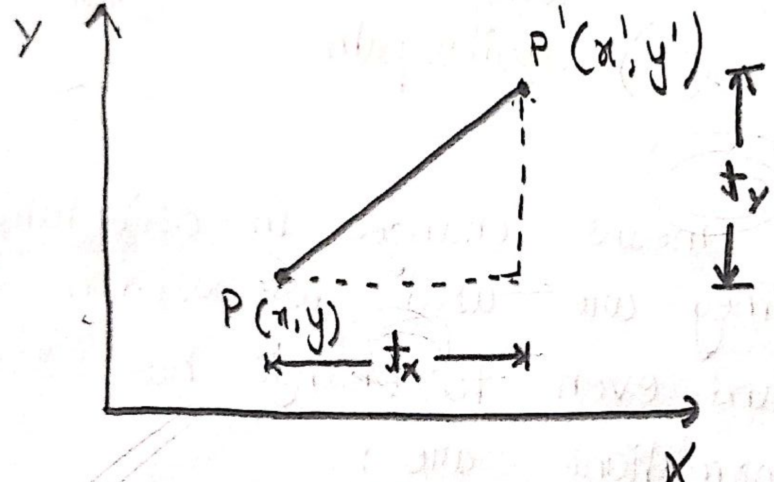
① Reflection
② Sheering

① **Translation** : → It is repositioning an object along straight line path from one coordinate location to another.

→ Translation is rigid body transformation that moves an object without deformation.

Q. कैसे करे Translation ?

$(t_x, t_y) \rightarrow$ Translation vector or shift vector.



$$\boxed{\begin{aligned} X' &= X + t_x \\ y' &= Y + t_y \end{aligned}}$$

The matrix representation will be -

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Q. Translate a point (2,4) where T(-1,14). Find P'.

$P \equiv (2,4)$

$T \equiv (-1,14)$

$P' = P + T$

$\boxed{P' = (1, 18)}$

$\times$ ⊕ $(-)$

Q. Translate a polygon with coordinates A(2,7), B(7,10), C(10,2] by 3 units in X direction and 4 units in Y direction.

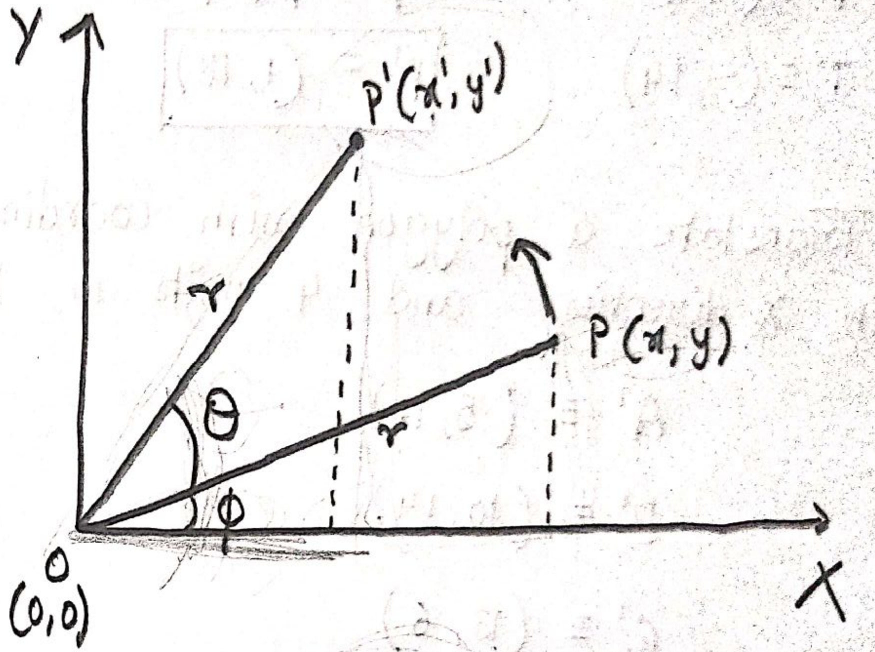
$A' \equiv (5, 11)$ ✓

$B' \equiv (10, 14)$ ✓

$C' \equiv (13, 6)$ ✓

② **Rotation** : Isme apn object को ek particular angle θ से rotate करते हैं from origin.

$$x' = r\cos(\theta+\phi)$$
$$= r(\cos\theta\cos\phi - \sin\theta\sin\phi)$$
$$= r\cos\theta\cos\phi - r\sin\theta\sin\phi$$
$$= x\cos\theta - y\sin\theta$$



$$y' = r\sin(\theta+\phi)$$
$$= r(\sin\theta\cos\phi + \cos\theta\sin\phi)$$
$$= r\sin\theta\cos\phi + r\cos\theta\sin\phi$$
$$= x\sin\theta + y\cos\theta$$

$x = r\cos\phi$

$y = r\sin\phi$

$$\boxed{\begin{aligned} x' &= x\cos\theta - y\sin\theta \\ y' &= x\sin\theta + y\cos\theta \end{aligned}}$$

Matrix Representation :

$$P' = \overset{\xrightarrow{\text{Rotation matrix}}}{R}\,P$$

$$\boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}}$$

Q. A point (4,3) is rotated in counter clock wise direction by the angle of 45°. Find the rotation matrix R and the resultant points.

$$P' = R P$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix}_{2\times2} \begin{bmatrix} 4 \\ 3 \end{bmatrix}_{2\times1}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}} \\ \frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}} \end{bmatrix}_{2\times1}$$

③ **Scaling** : → A scaling transformation alters the size of an object.

→ Scaling of a polygon requires multiplying the coordinate value of each vertex by the scaling factor to get the new coordinate value.

$$\boxed{\begin{aligned} x' &= x\, S_x \\ y' &= y\, S_y \end{aligned}}$$

$0 < S_x, S_y < 1 \rightarrow$ size ↓

$S_x, S_y > 1 \longrightarrow$ size ↑

$S_x = S_y \longrightarrow$ uniform scaling

In matrix form, it can be represented as :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = S\,P$$

Q. Translate a square ABCD, A(0,0), B(3,0), C(3,3) & D(0,3) by 2 units in both the direction and then scale it by 1.5 units in X-direction and 0.5 units in Y-direction. Determine the resultant coordinates of polygon.

→ Translation :

A' (2, 2) B'(5, 2) C' (5, 5) D' (2, 5).

→ Scaling :

A''(3, 1) B''(7.5, 1) C''(7.5, 2.5). D''(3, 2.5)

Q. Magnify the Δ^{e} with vertices (0,0), (1, 1), (5, 2) to twice its size while keeping (5, 2) fixed.

$\Delta^{le} \xrightarrow{\text{Scale}} \Delta^{le} \xleftarrow{\text{translate}} \Delta^{le}$

→ Scaling: A'(0,0) B'(2,2) C'(10, 4)

→ Translate : A''(-5, -2) B''(-3, 0) C''(5, 2)

Q. Scale a polygon with coordinates A (2, 5), B (7, 10), C (10, 2) by 2 units in X-direction & 3 units in Y-direction.

$S_x = 2$ $S_y = 3$

→ A (2, 5)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \end{bmatrix}$$

⇒ A'(4, 15)

→ B(7, 10)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 10 \end{bmatrix} = \begin{bmatrix} 14 \\ 30 \end{bmatrix}$$

⇒ B'(14, 30)

→ C (10, 2)

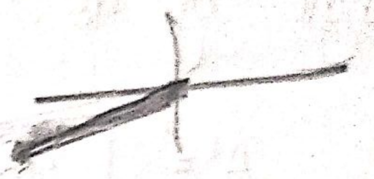
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 20 \\ 6 \end{bmatrix}$$

⇒ C' (20, 6)

Q. Rotate the same Δ^{le} about the pivot point $(-2, -2)$.

$$\begin{tikzcd}
\Delta^{le} \arrow[r, "T"] & \Delta^{le} \arrow[r, "R"] & \Delta^{le} \arrow[r, "T"] & \Delta^{le}
\end{tikzcd}$$



Translate : A $(2, 2)$ B $(4, 4)$ C $(6, 4)$

$2-(-2) = 4$

Rotate :

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2\sqrt{2} \end{bmatrix} \quad A'$$

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 4\sqrt{2} \end{bmatrix} \quad B'$$

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} \frac{6}{\sqrt{2}} - \frac{4}{\sqrt{2}} = \sqrt{2} \\ \frac{6}{\sqrt{2}} + \frac{4}{\sqrt{2}} = 5\sqrt{2} \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ 5\sqrt{2} \end{bmatrix} \quad C'$$

Translate : $A''(-2, 2\sqrt{2} - 2)$. $B''(-2, 4\sqrt{2} - 2)$ $C''(\sqrt{2} - 2, 5\sqrt{2} - 2)$

Q. Rotate a Δ^{le} A (0,0) B (2,2) C(4,2) about the origin by an angle of 45°.

$$A' = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ ✓

$$B' = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4\sqrt{2} \end{bmatrix}$$ ✓

$$C' = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 2\sqrt{2} \\ 3\sqrt{2} \end{bmatrix}$$ ✓

Homogenous Coordinate System :

→ A homogenous coordinate system is an abstract representation technique in which we represent a 2-D point $P(x, y)$ with a 3-element vector $P_h(xh, yh, h)$ with the relationship :

$$x = \frac{xh}{h} \quad , \quad y = \frac{yh}{h}$$

The term h is the homogeneous factor & can take any non-zero value.

For eg: point (3, 4) has homogenous coordinates (6, 8, 2).

* जब हम kisi coordinate को Translate, Rotate, Scale करते हैं तो process में Time lgta है तो isi process को jldi se krne के लिए instead of 2x2 matrix we use 3x3 matrix.

* Isi के लिए apne ko h (Dumy coordinate) : को Use करना पड़ता है।

* In this system, we can represent all the transformation equations in a matrix multiplication form.

① Translation :

$x' = x + t_x$
$y' = y + t_y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = T(t_x, t_y) \cdot P$$

② Rotation :

$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

✯ In homogenous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

③ Scaling :

$$X' = X \delta_x \qquad \Rightarrow \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \delta_x & 0 \\ 0 & \delta_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

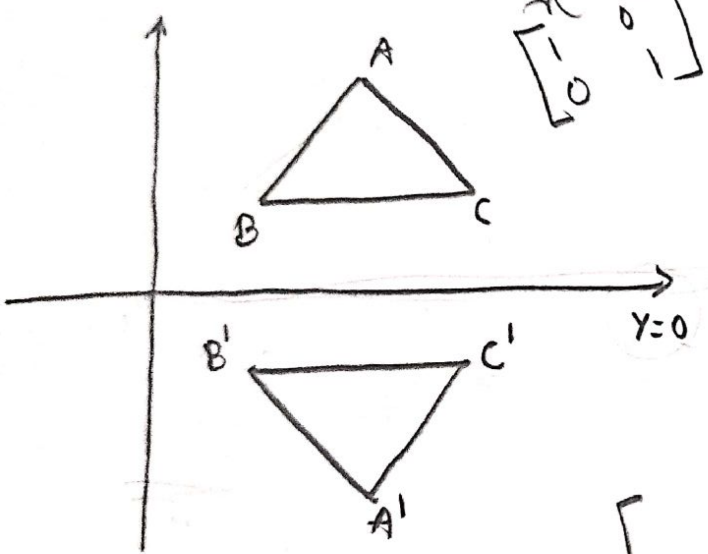
$$Y' = Y \delta_y$$

✯ In homogenous coordinates:

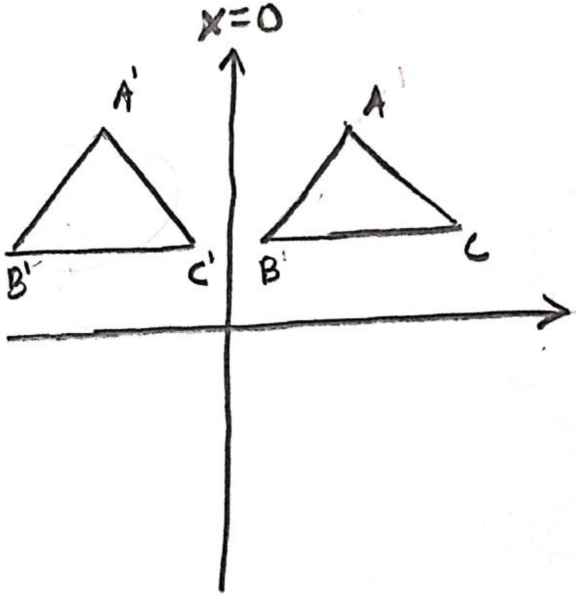
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \delta_x & 0 & 0 \\ 0 & \delta_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

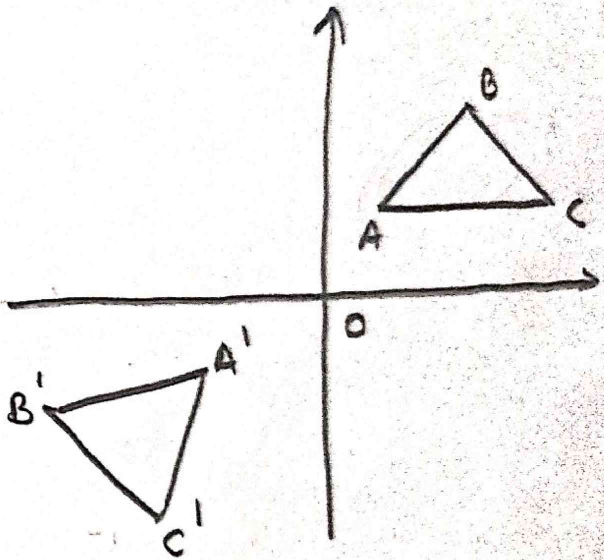
Reflection : → A reflection is a transformation that produces a mirror image of an object.

→ The mirror image of 2-D reflection is generated relative to an axis of reflection by rotating the object 180° to the reflection axis.



$$\begin{array}{cc} x & y \\ \left[\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}\right] \end{array}$$



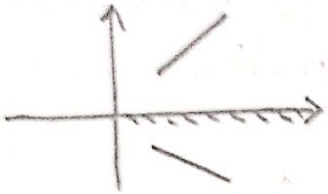


$$\begin{array}{ccc} x & y & \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{array}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

① About X-axis :



$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

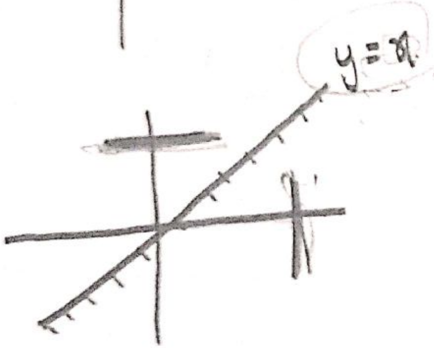
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

② About Y-axis :

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

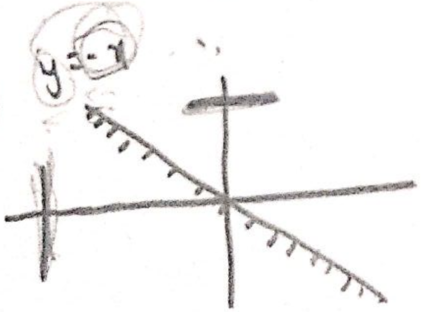
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

③



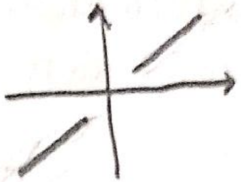
$$\begin{array}{cc} x & y \\ \left[\begin{matrix} 0 \\ 1 \end{matrix}\right. & \left.\begin{matrix} 1 \\ 0 \end{matrix}\right] \end{array}$$

④



$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

⑤ About origin



$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Q. Determine the transformation matrix of a Δ^{le} A (4, 1) . B (5, 2) , C (4, 3) about the line $x = 0$ and determine the resultant coordinates.

About Y-axis :

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A' (−4, 1)

B' (−5, 2)

C' (−4, 3)

$$A' \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$$

Shearing : → The shearing transformation distorts the shape of the object.
↓
Skewing

→ There are 2 types of shearing transformation:

(i) X - shearing

(ii) Y - shearing

(i) X-shearing : → It preserves the y coordinates but changes the x values.

→ For any point $P(x,y)$

$P'(x+shx \cdot y \ , y)$, where shx is the shearing vector in x direct

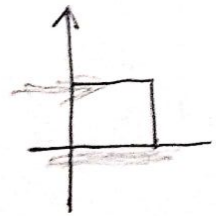
(ii) Y-shearing : → The x coordinate remains same while there is a change in y value.

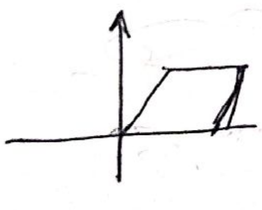
$x+Shx\,y$

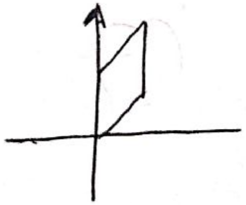
$y+Shy \cdot x$

→ For any point $P(x, y)$

$P'(x, y+shy \cdot x)$, where shy is the shearing vector in y direction.







Shearing:

X-shearing :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Y-shearing:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Reflection:

(i) About X-axis :

$$\begin{bmatrix} X' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(ii) About origin :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(iii) About Y axis :

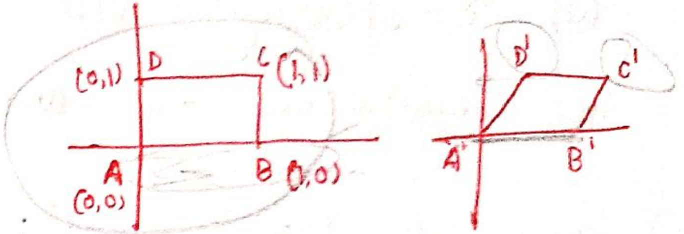
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(iv) About $y = x$:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Q. Shear a polygon A (0,0) B (1,0) C(1,1) D (0,1) by shearing vector $sh_x=2$ and determine the new Coordinates.

A(0,0)	B (1,0)	C (1,1)	D (0,1)
A' (0,0)	B' (1,0)	C' (3,1)	D' (2,1)



☆ When shearing is applied on both the direction simultaneously, then the matrix will be :

$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

When $b=0 \rightarrow$ x Shearing

$a=0 \rightarrow$ y Shearing

Q. A polygon with vertices A(0,0) B(1,0) C(1,1) D(0,1). Perform the following transformations and conclude the result of shearing.

(a) x-shearing with a=2 followed by y-shearing with b=3

(b) Simultaneous x and y shearing with a=2 and b=3.

(a) A' $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

A' (0,0)

B' $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

B'(1,0)

D' (2,1)

C' $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

(b) $a=2, \quad b=3$ $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

A' $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $A'(0,0)$

B' $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ $B'(1,3)$

C' $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$ C' $(3,4)$

D' $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ $D'(2,1)$

Now, y Shearing : $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A'' \begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad A''(0,0)$$

$$B'' \begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \qquad B''(1,3)$$

$$C'' \begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix} \qquad C''(3,10)$$

$$D'' \begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix} \qquad D''(2,7)$$

Q. Show that reflection about the line $y = -x$ is equivalent to reflection relative to y-axis followed by counter clockwise rotation of 90°.

→ The matrix for the reflection $y = -x$ is $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

→ We need to proove that this is equivalent to a matrix obtain by reflection about the y-axis followed by 90° rotation in counterclockwise direction.

→ The composite matrix for this transformation is –

$$T_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T_2 = \begin{bmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The composite transformation matrix will be :

$$T_2 \cdot T_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This proves that the required condition is true.

Q. Δ^{le} ABC where A(-1,-3), B(-4,-1), C(-6,-4) undergoes a composition of transformation described as:

(a) A translation of 10 units to the right

(b) A reflection in x-axis

What are the vertices of Δ^{le} after both the transformation.

$\rightarrow t_x = 10 \Rightarrow T_1 = \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$T_2 \cdot T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 10 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0. & 10 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 1 \end{bmatrix}$$

A'(9, 3)

C(4,4)

$$B' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 10 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix}$$

B'(6,1)

3-D Transformation:

→ In 3D we have 3 axis x, y, z such that theser are normal to each other.

① 3-D Translation:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

② 3-D Scaling:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \delta_x & 0 & 0 & 0 \\ 0 & \delta_y & 0 & 0 \\ 0 & 0 & \delta_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

③ 3-D Rotation :

✯ Rotation about x axis :

$$R_x = \begin{array}{c} \begin{matrix} x & y & z & \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

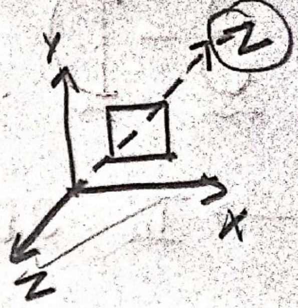
✯ Rotation about y axis :

$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

✯ Rotation about z axis :

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

④ 3-D Reflection



✯ In X-Y plane

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

✯ In Y-Z plane

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

✯ In Z-X plane

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Q. For the given matrix, first apply a rotation of 45° about the y-axis followed by rotation of 45° about x-axis and determine the resultant matrix?

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 0 & 3 & 6 & 1 \end{bmatrix}$$

→ There are two rotations R_y & R_x.

$$R_y = \begin{bmatrix} \cos 45° & 0 & \sin 45° & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 45° & 0 & \cos 45° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 45° & -\sin 45° & 0 \\ 0 & \sin 45° & \cos 45° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, Composite transformation.

$$R = \underline{R_x \cdot R_y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The resultant matrix will be -

$$RM = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 0 & 3 & 6 & 1 \end{bmatrix}$$

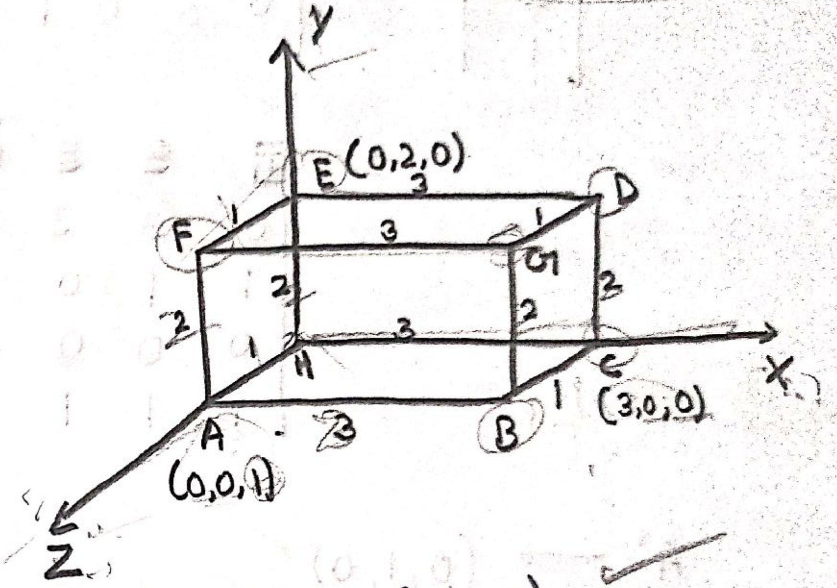
$$= \begin{bmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & 3/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q. A rectangular parallelopiped has its length as 3 unit, 2unit & 1 unit on X, Y & Z-axis respectively. Perform rotation by 90° clockwise about X-axis.

→ Rotation about X-axis :

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-90^\circ) & -\sin(-90^\circ) & 0 \\ 0 & \sin(-90^\circ) & \cos(-90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



A → (0,0,1)
B → (3,0,1)
C → (3,0,0)
D → (3,2,0)
E → (0,2,0)
F → (0,2,1)
G → (3,2,1)
H → (0,0,0)

✯ Transformation matrix :

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4} \begin{bmatrix} 0 & 3 & 3 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 & 2 & 2 & 2 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}_{4\times8}$$

$$= \begin{bmatrix} 0 & 3 & 3 & 3 & 0 & 0 & 3 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & -2 & -2 & -2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}_{4\times8}$$

A' ⟶ (0, 1, 0)

B' ⟶ (3, 1, 0)

C' ⟶ (3, 0, 0)

D' ⟶ (3, 0, −2)

E' ⟶ (0, 0, −2)

F' ⟶ (0, 1, −2)

G' ⟶ (3, 1, −2)

H ⟶ (0, 0, 0)

(5)

$O(n^2)$

$5 \log 5$

$(5)^{3/2}$

$(125)^{1/2}$

2-D Viewing

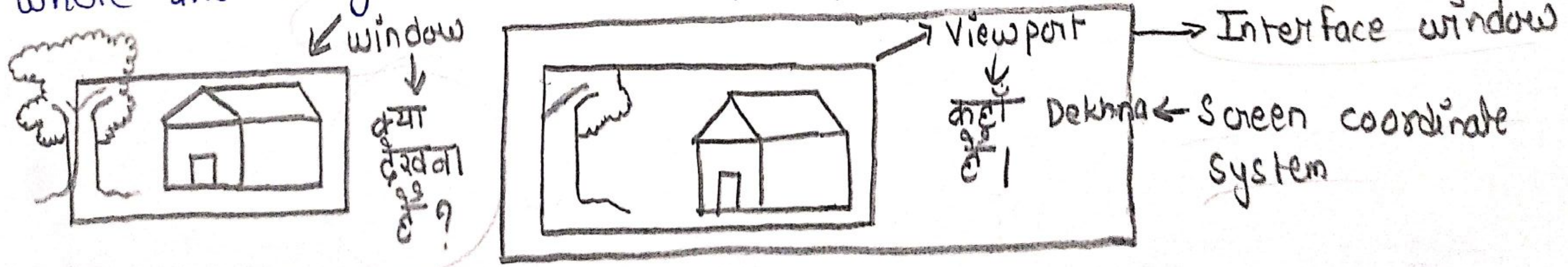
→ A graphics package that allows a user to specify which part of a defined picture is to be displayed and where that part is to be displayed on the display device using a concept known as clipping.

→ World coordinate System : This is the object space or the space in which application model is defined.

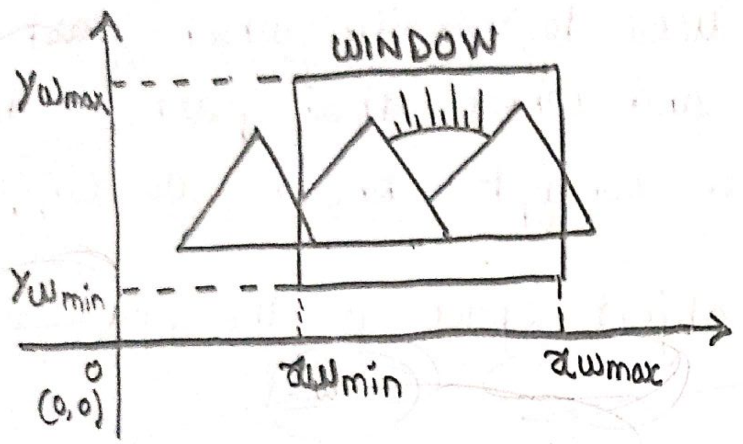
→ Screen coordinate system : The space in which the image is displayed is called as screen coordinate system.

* Window : The method of selecting the portion of the drawing is called windowing & the rectangular area which is seleted is called window.

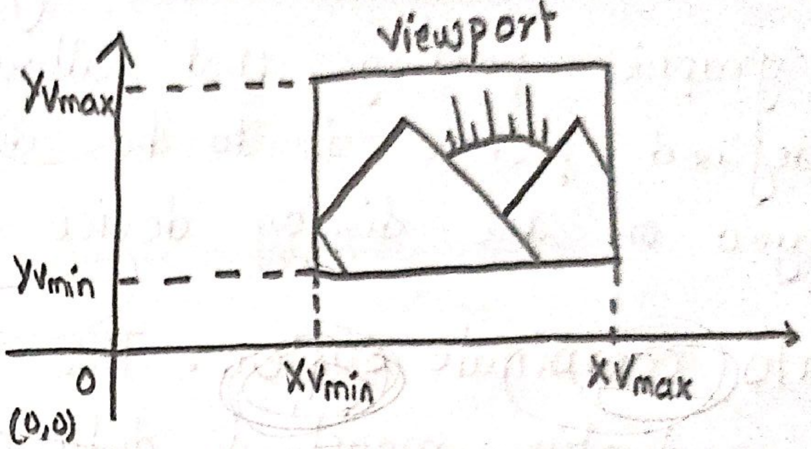
* Viewport : The rectangular portion of the interface window that defines where the image which actually appear is called viewport.



Viewing Pipeline :



World coordinates.

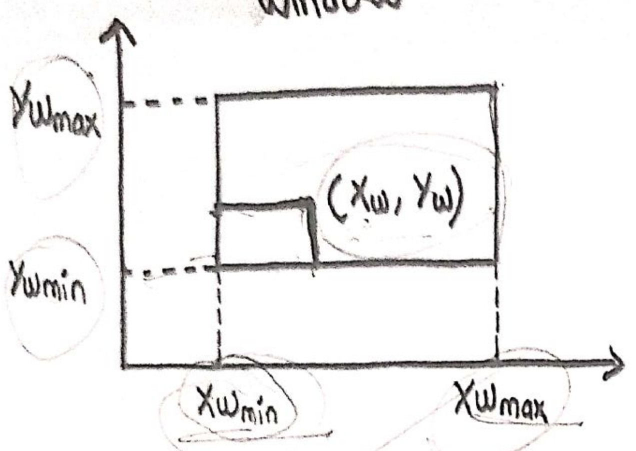


Device coordinates

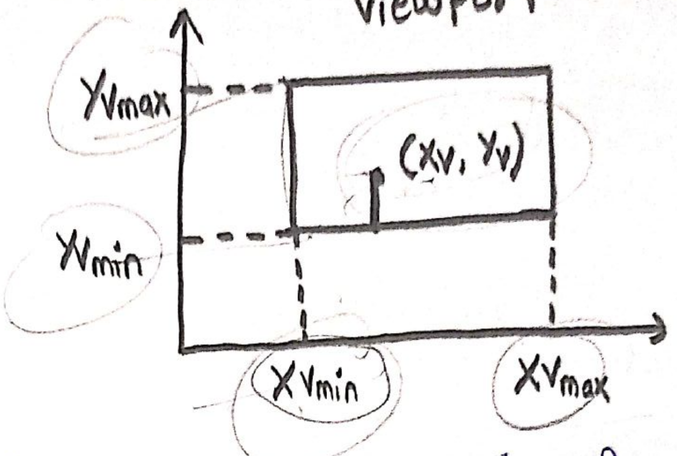
- ✡ Mapping of a part of a world coordinate scene to device coordinate is referred to as Viewing Transformation

- ✡ 2-D viewing transformation is simply referred to as the "Window to viewport" transformation or "Windowing Transformation

Window



Viewport



* बस एक बात याद रखना है "Relative position" नही Badlegi!

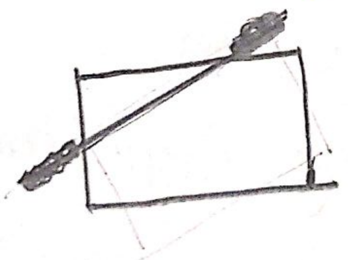
$$\frac{x_w - x_{w_{min}}}{x_{w_{max}} - x_{w_{min}}} = \frac{x_v - x_{v_{min}}}{x_{v_{max}} - x_{v_{min}}} \Rightarrow \boxed{x_v = x_{v_{min}} + (x_w - x_{w_{min}}) \delta_x}$$

δ_x, δ_y → Scaling factor

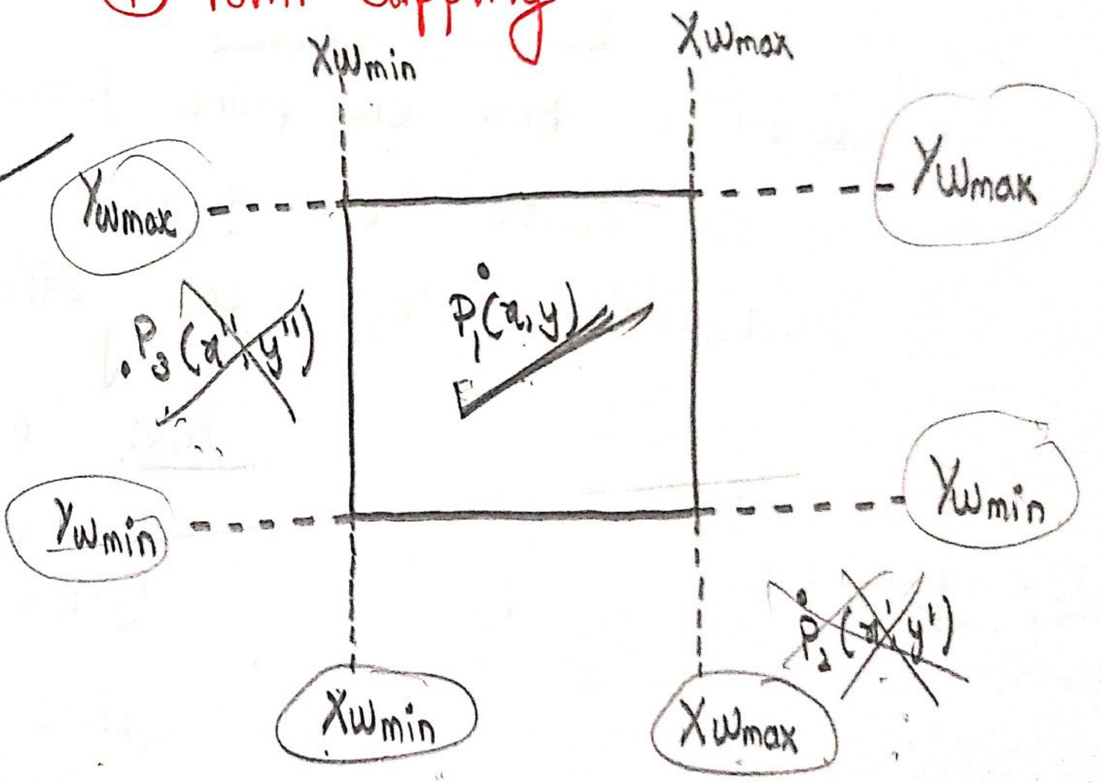
$$\frac{Y_w - Y_{w_{min}}}{Y_{w_{max}} - Y_{w_{min}}} = \frac{Y_v - Y_{v_{min}}}{Y_{v_{max}} - Y_{v_{min}}} \Rightarrow \boxed{Y_v = Y_{v_{min}} + (Y_w - Y_{w_{min}}) \delta_y}$$

Clipping → जो Portion window के बाहर होता है, उसे clipped part bolte है और the process of displaying inside image of the window is called clipping.

Types :
- → Point clipping
- → Line clipping
- → Polygon clipping
- → Curve clipping
- → Text clipping



① Point clipping

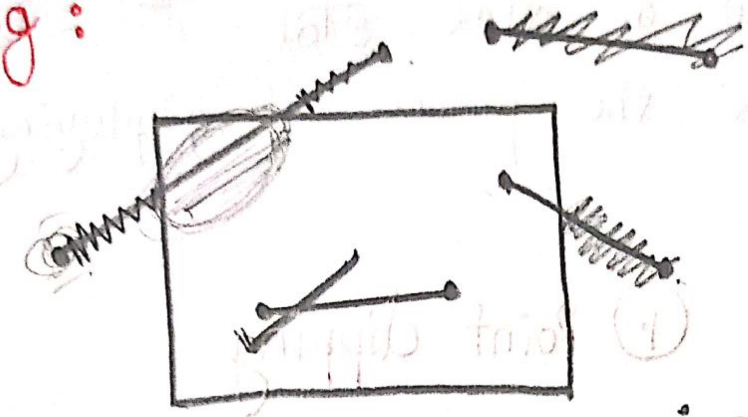


→ कोई Point P(x,y) will be displayed if:

$Xw_{min} \leq X \leq Xw_{max}$

$Yw_{min} \leq Y \leq Yw_{max}$

② Line clipping :



जो window के बाहर है, उसको remove करो।

(i) visible : Both end points inside हो।

(ii) NOT visible : Both points outside हो

(iii) Partially visible : ½ andar और ½ bahar !

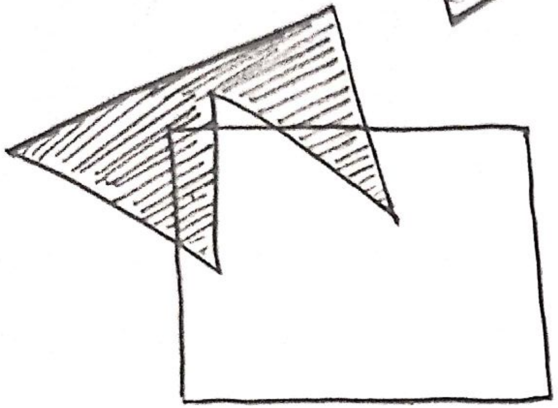
↓

कैसे करे Clipping

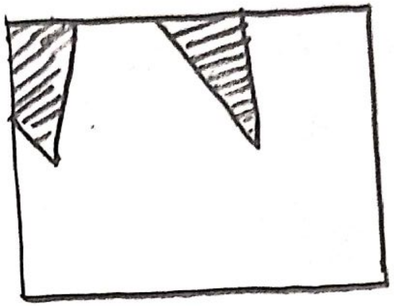
↓

"Cohen Sutherland Line Clipping Algorithm"

③ Polygon Clipping: → Sutherland Hodgeman Algorithm.

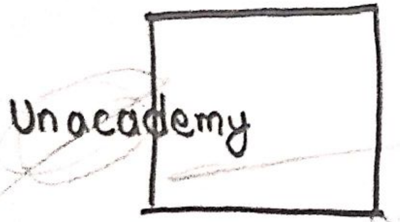


Before Clipping

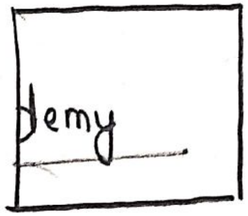


After clipping

④ Text Clipping :

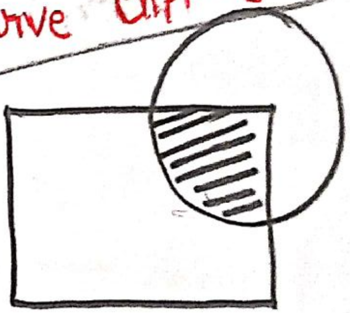


Before



After

⑤ Curve Clipping :



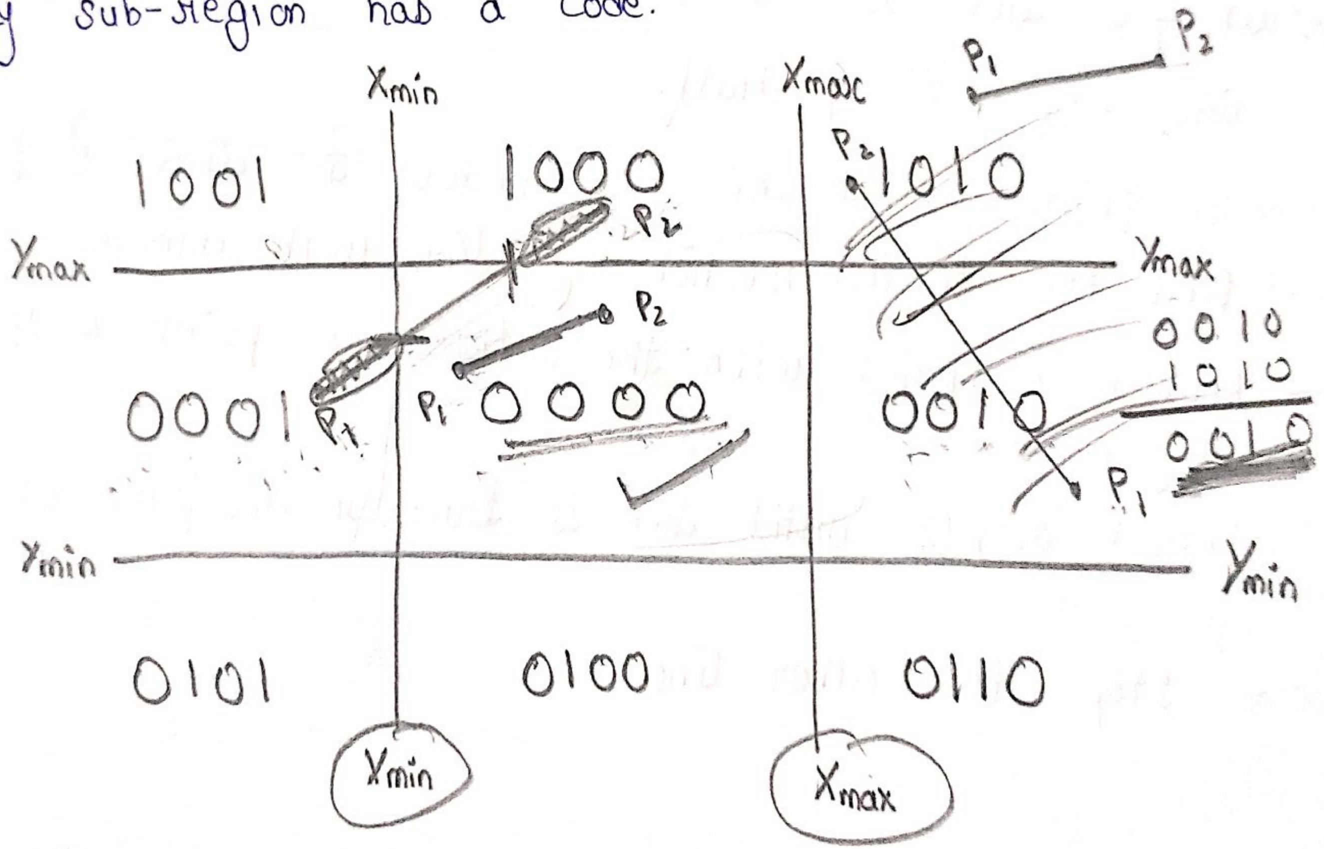
Before

After

Cohen Sutherland Line Clipping Algorithm:

✱ Determining the visibility of the line :

→ In Cohen Sutherland algorithm outcode or region code are used to determine the visibility of the line, the whole region is divided into 9 Subregions and every Sub-region has a code.



TBRL

- → X < Xmin Ⓛ
- → X > Xmax Ⓡ
- → Y < Ymin Ⓑ
- → Y > Ymax Ⓣ

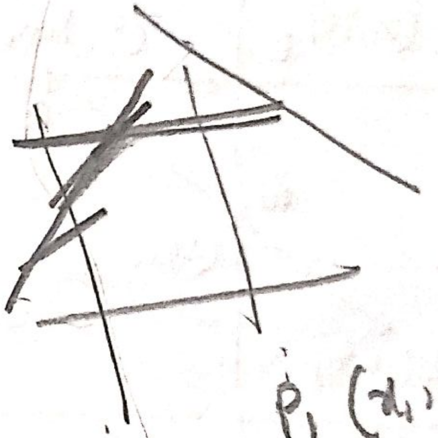
~~ALGORITHM~~ Pseudo Code :

```
1. Assign the region code for two end points of a given line.
2. If both have region codes 0000 then line accepted completely
3. else perform logical AND operation for both region codes.
   (a) if result ≠ 0 line is बाहर।
   (b) else line अंदर है partially
       → ek point jo line का jo window के बाहर है।
       → find the intersection of line with window.
       → Replace endpoint with the intersection point & update
         region code.
       → Repeat step (2) until line is trivially accepted or rejected.

4. Repeat above steps for other lines.
```

ALGORITHM :

```
① Assign Region code to both end points (Let it be ⓒ₀ & C₁)
② If C₀ or C₁ == 0000 (then completely accepted) → मतलब window के अंदर !
   else if
        C₀ && C₁ ≠ 0000    Reject it
   else
      Clip if line crosses $X_{min}$ or $X_{max}$
         then
              $y = y_1 + m(x - x_1)$
         else
              $x = x_1 + \frac{1}{m}(y - y_1)$
③ Verify    $X_{min} \le X \le X_{max}$
            $Y_{min} \le Y \le Y_{max}$
   if it does not satisfy then repeat
```

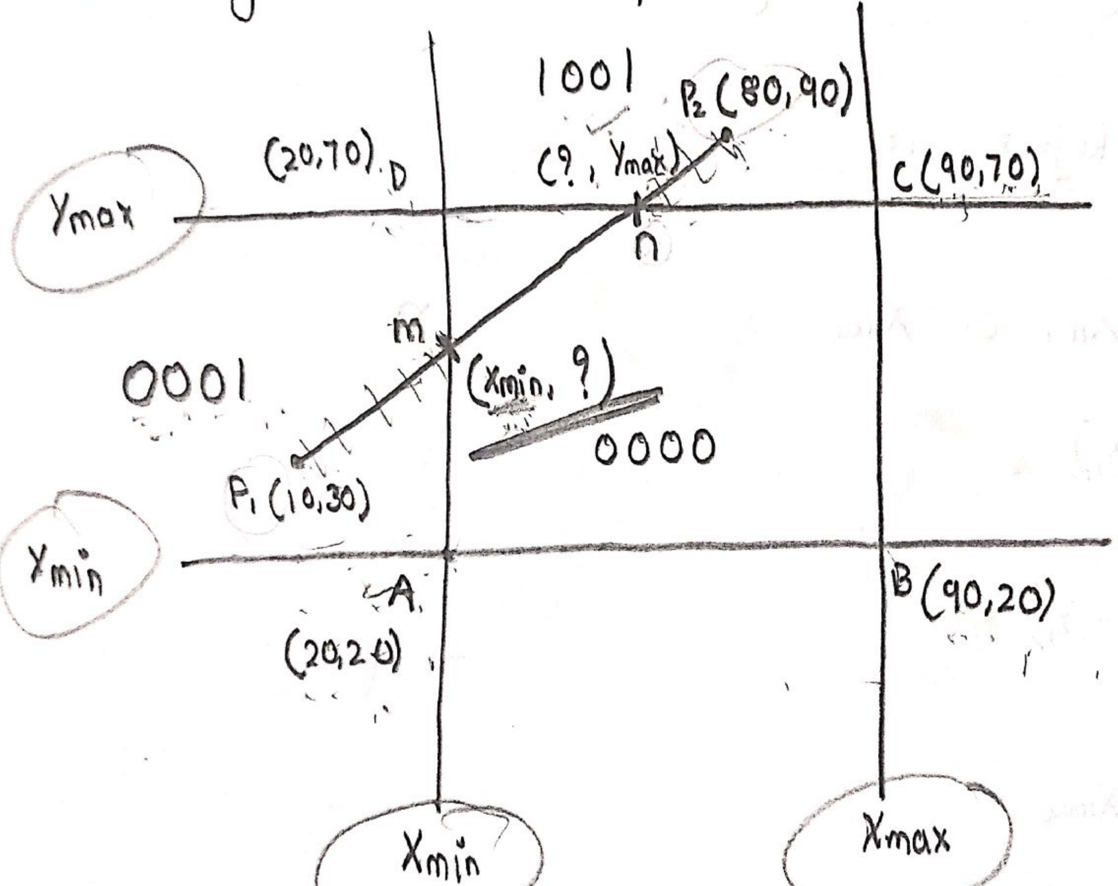


$P_1(x_1, y_1)$

$P_2(x_2, y_2)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Q. Let ABCD be the rectangular window with A(20,20), B(90,20), C(90,70) and D(20,70). Find region codes for the end points and use Cohen-Sutherland algorithm to clip the line P_1P_2 with $P_1(10,30)$ & $P_2(80,90)$.



① Finding Region code :

$P_1(10, 30) \rightarrow X_1, Y_1$

$P_2(80, 90) \rightarrow X_2, Y_2$

$P_1 = 0001$

$\odot \; P_2 = 1000$

$\overline{0000} \rightarrow$ Line is partially visible. (Clip करो isko)

② Slope, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90-30}{80-10} = \frac{6}{7} = 0.857$

→ For point of intersection ⓜ :-

$y = m(X_{min} - X_1) + Y_1$

$= 0.857(20 - 10) + 30$

$= 8.57 + 30$

$= 38.57$

≈ 39

$\rightarrow \boxed{m(20, 39)}$

$m(20, 39)$

→ For point n :

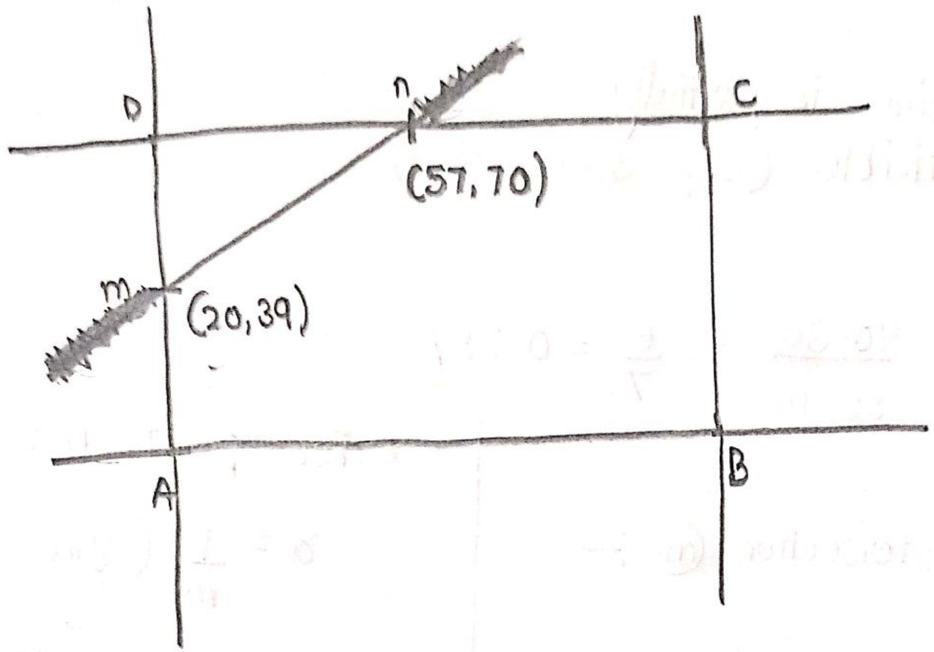
$x = \frac{1}{m}(y_{max} - y_1) + x_1$

$= \frac{1}{0.857}(70 - 30) + 10$

$= 56.67$

≈ 57

$\rightarrow \boxed{n(57, 70)}$

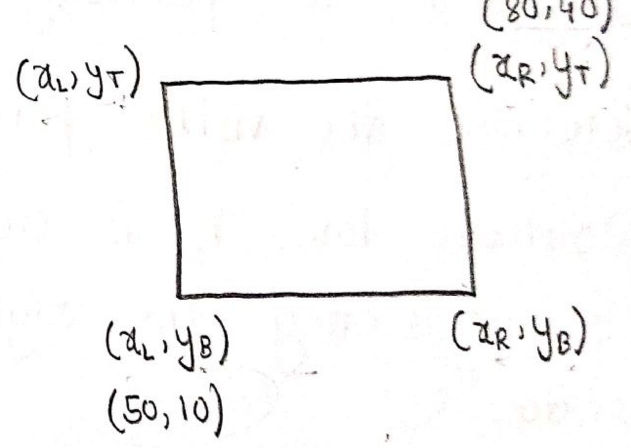

D
n
C
(57,70)
m
(20,39)
A
B

→ Formula to calculate the bit code :

- ✡ Bit 1 = Sign of $(y - y_T)$
- ✡ Bit 2 = Sign of $(y_B - y)$
- ✡ Bit 3 = Sign of $(x - x_R)$
- ✡ Bit 4 = Sign of $(x_L - x)$

* Sign = 1 → +ve
* Sign = 0 → −ve

Q. Use the cohen sutherland algorithm to determine the visibility of a line P_1P_2 $P_1(\overset{x}{70}, \overset{y}{20})$, $P_2(100, 10)$ against a window with lower left hand corner at $(50, 10)$ and upper right hand corner at $(80, 40)$.



Outcode for P_1 :

Bit 1 = $S(20 - 40) = -20 = 0$

Bit 2 = $S(10 - 20) = -10 = 0$

Bit 3 = $S(70 - 80) = -10 = 0$

Bit 4 = $S(50 - 70) = -10 = 0$

Outcode for P2 : $(\overset{x}{100},\overset{y}{10})$

Bit 1 = $s(10-40) = s(-30) = 0$

Bit 2 = $s(10-10) = 0$

Bit 3 = $s(100-80) = 1$

Bit 4 = $s(50-100) = s(-50) = 0$

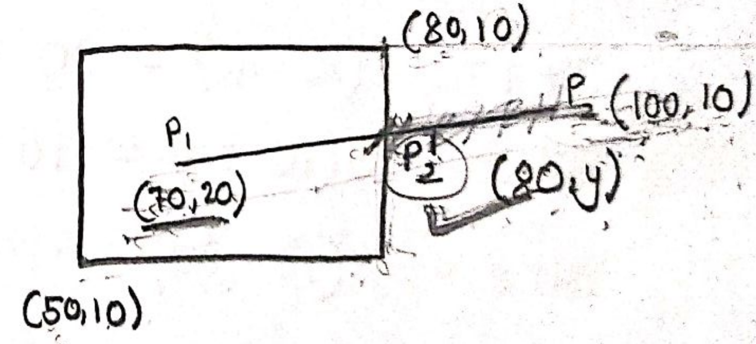
As, the outcode for both the end point is not 0000.

∴ the line is partially visible (0000 AND 0010 = 0000)

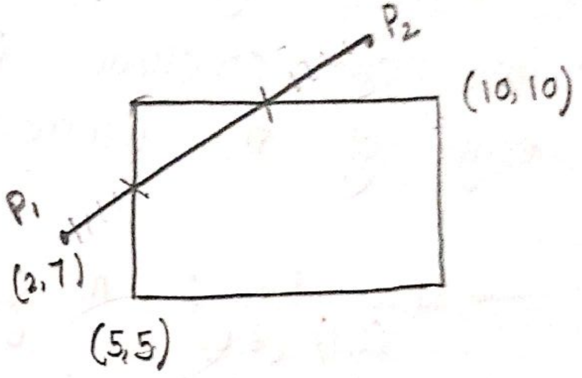
→ Now determine the visible portion of this line :

As, the outcode for P_2 is $\overset{TBRL}{0010}$

∴ It is intersecting the right edge of the window.



Q. Use the clipping algorithm for calculating. The visible portion of the line $P_1(2, 7)$, $P_2(8, 12)$ against a window where $(X_{min}, Y_{min}) = (5, 5)$ & $(X_{max}, Y_{max}) = (10, 10)$



$X_L = 5$, $X_R = 10$, $Y_B = 5$, $Y_T = 10$

→ Bit code $P_1(2, 7)$

$B_1 = 7-10 = 0$

$B_2 = 5-7 = 0$

$B_3 = 2-10 = 0$

$B_4 = 5-2 = \textcircled{1}$

→ Bit code $P_2(8, 12)$

$B_1 = 12-10 = 1$

$B_2 = 5-12 = 0$

$B_3 = 8-10 = 0$

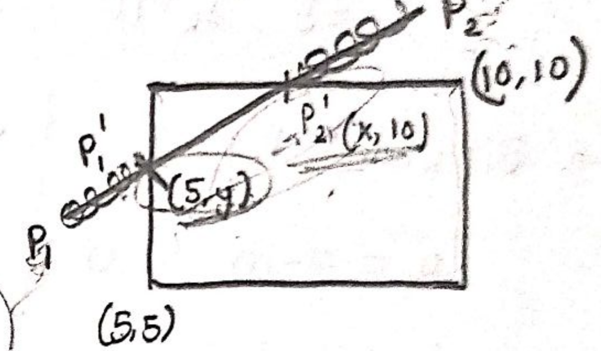
$B_4 = 5-8 = 0$

→ Perform AND operation:

$$\begin{array}{r} 0\,0\,0\,1 \\ \odot\ 1\,0\,0\,0 \\ \hline 0\,0\,0\,0 \end{array} \longrightarrow \text{Line is partially visible}$$

→ As, a result of ANDing operation for bitcode of both the end point is 0000. ∴ the line is partially visible and need to be clipped.

→ The outcode of P_1 is 0001.
∴ we can conclude that the line is extending (0001) left edge of the window.



→ Line P_1P_2 : $m = \frac{5}{6}$ ✓
(2,7) (8,12)

→ Line $P_1'P_2$: $m = \frac{12-y}{3} = \frac{5}{6} \Rightarrow \boxed{y=9.5}$
(5,y) (8,12)

→ Now, also finding outcode of $P_1'(\overset{x}{5}, \overset{y}{9.5})$ ✓

$B_1 = 9.5 - 10 = 0$
$B_2 = 5 - 9.5 = 0$
$B_3 = 5 - 10 = 0$
$B_4 = 5 - 5 = 0$

As, the bitcode of P_1' is 0000.
∴ It is inside the clipping window.

$P_1'P_2'$ $m = \frac{10-9.5}{x-5} = \frac{5}{6} \Rightarrow \boxed{x = 5.6}$

Finding out outcode of P_2' (5.6, 10)
$B_1 = 10-10=0$, $B_2 = 5-10=0$, $B_3 = 5.6-10=0$,
$B_4 = 5-5.6 = 0$

As, outcode of both the end points P_1', P_2' is 0000. ∴ Line is completely inside the clipping window. & visible portion is (5, 9.5) & (5.6, 10)

→ Determining the intersection point P_2' :

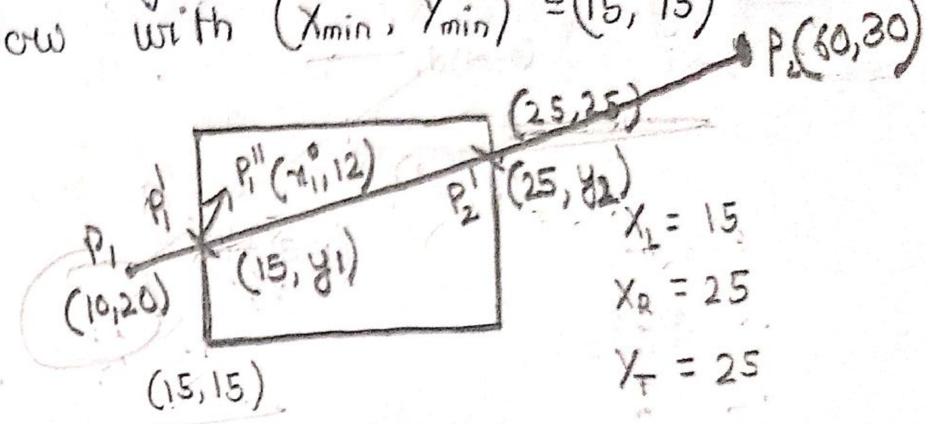
Slope of the line $P_1P_2 = \frac{10-20}{100-70} = \frac{-10}{30} = -\frac{1}{3}$

* $P_2'P_1$:-

$$-\frac{1}{3} = \frac{y-20}{80-70} \Rightarrow \boxed{y = \frac{50}{3}}$$

→ P_1P_2' is the visible portion of line with coordinates (70,20) & (80, 16·67).

Q. Find out the coordinates of the clipped line for a line P_1P_2 where $P_1(10,20)$ & $P_2(60,30)$ against a window with $(X_{min}, Y_{min}) = (15,15)$ and $(X_{max}, Y_{max}) = (25,25)$.



$X_L = 15$

$X_R = 25$

$Y_T = 25$

$Y_B = 15$

→ $P_1(10,20)$: ✓

$B_1 = 10-25 = 0$ T

$B_2 = 15-10 = 1$ (B)

$B_3 = 10-25 = 0$ R

$B_4 = 15-10 = 1$ (L)

→ $P_2(60,30)$:

$B_1 = 60-25 = 1$ T ✓

$B_2 = 15-60 = 0$ B

$B_3 = 30-25 = 1$ R ✓

$B_4 = 15-30 = 0$ L

$$\begin{array}{r} 0101 \\ 1010 \\ \hline \text{AND } 0000 \end{array}$$

∴ Partially visible

→ For point P_1

$$m = \frac{30-10}{60-10} = \frac{20}{50} = \frac{2}{5}$$

$P_1' P_2$:

$$\frac{30-y_1}{45} = \frac{2}{5} \Rightarrow \boxed{y_1 = 12}$$

$$\boxed{P_1'(15, 12)}$$

$B_1 = 12-25 = 0$

$B_2 = 15-12 = 1$

$B_3 = 15-25 = 0$

$B_4 = 15-15 = 0$

$P_1''(x_1, 15)$

$P_1'' P_2$

$$\frac{15}{60-x_1} = \frac{2}{5}$$

$x_1 = 22.5$

$$\boxed{P_1''(22.5, 15)}$$

$B_1 = 15-25 = 0$

$B_2 = 15-15 = 0$

$B_3 = 22.5-25 = 0$

$B_4 = 15-22.5 = 0$

P_2' $(25, y_2)$

P_1'' $(22 \cdot 5, 15)$.

→ $P_1'' P_2'$:

$$\frac{y_2 - 15}{2 \cdot 5} = \frac{2}{5}$$

$$\boxed{y_2 = 16}$$

$\therefore P_2'(25, 16)$,

$B_1 = 16 - 25 = 0$

$B_2 = 15 - 16 = 0$

$B_3 = 25 - 25 = 0$

$B_4 = 15 - 25 = 0$

As, the outcode for $P_1'' P_2'$ is 0000.

$\therefore$ The line is completely visible,

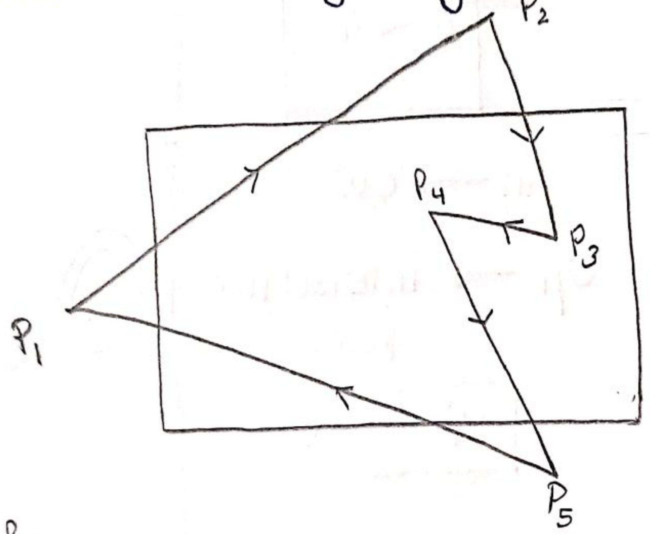
& the visible portion is $(22 \cdot 5, 15)$

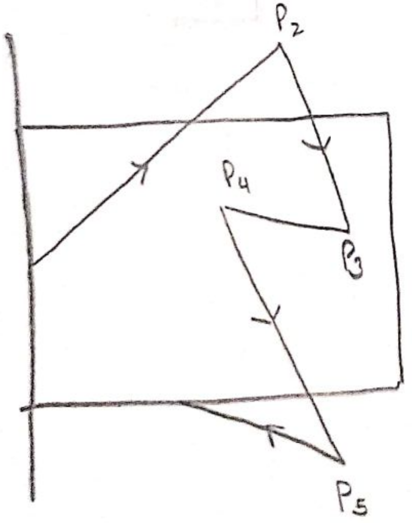
& $(25, 16)$.

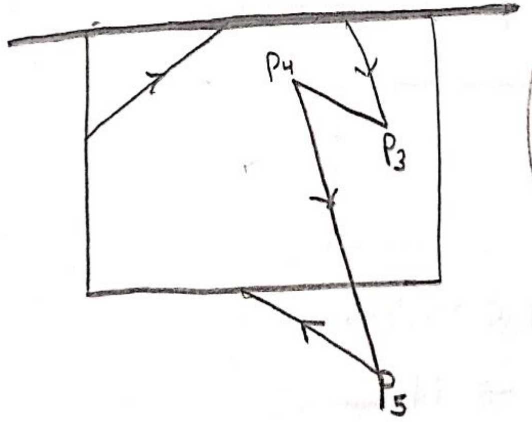
Sutherland - Hodgeman Polygon Clipping Algorithm

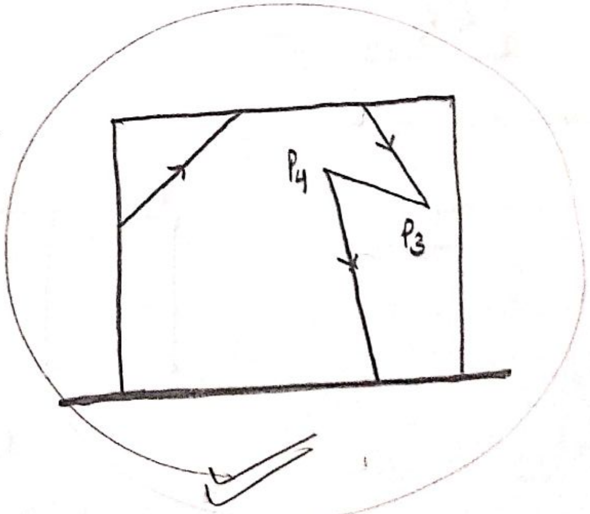
→ It clips the region of the polygon lying outside the window.

→ It clips against each edge of window & obtain new set of vertices.



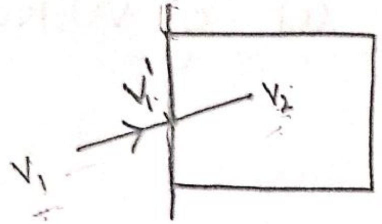






Rules for clipping the polygon edge :

①

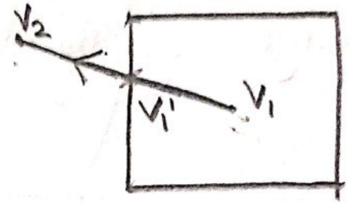


out → in

O/P → intersection point + destination point

$\boxed{V_1' V_2}$

②

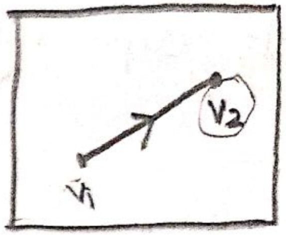


in → out

O/P → intersection point

$\boxed{V_1'}$

③

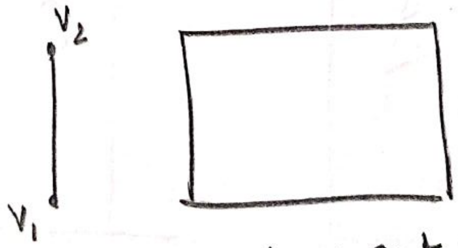


in → in

O/P → destination point

$\boxed{V_2}$.

④

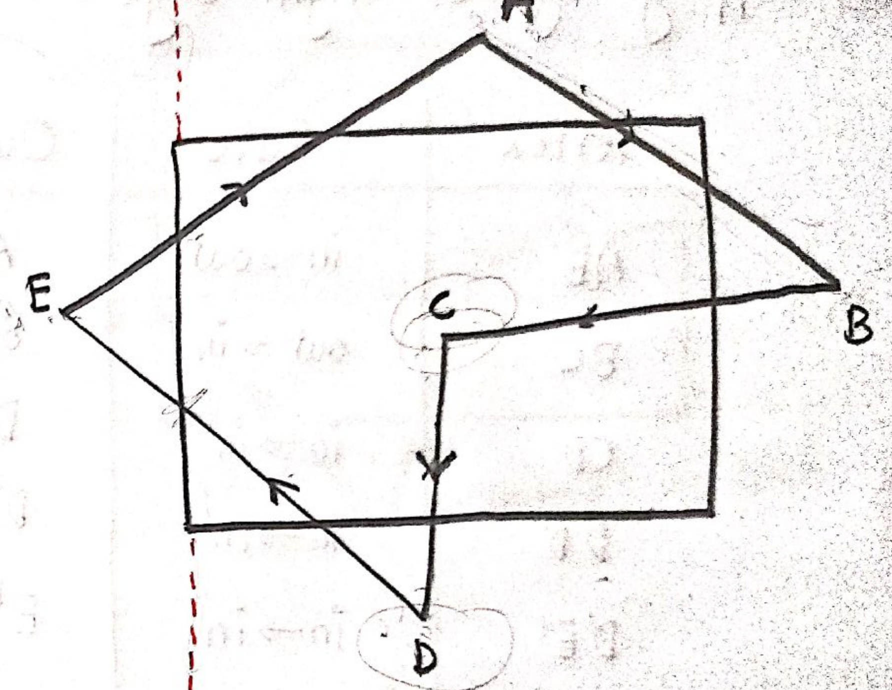


out → out

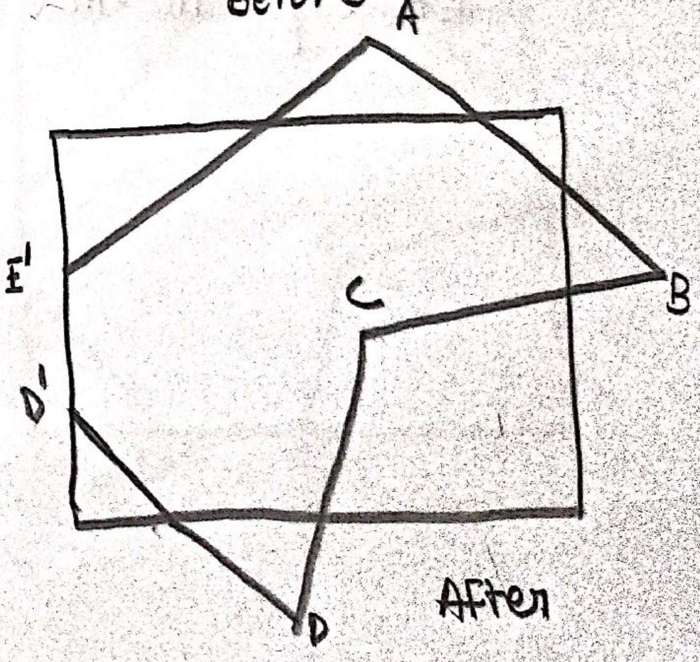
O/P → NULL

① Clipping against left edge :

Vertex	Rule	Output
AB	in → in	B
BC	in → in	C
CD	in → in	D
DE	in → out	D'
EA	out → in	E'A



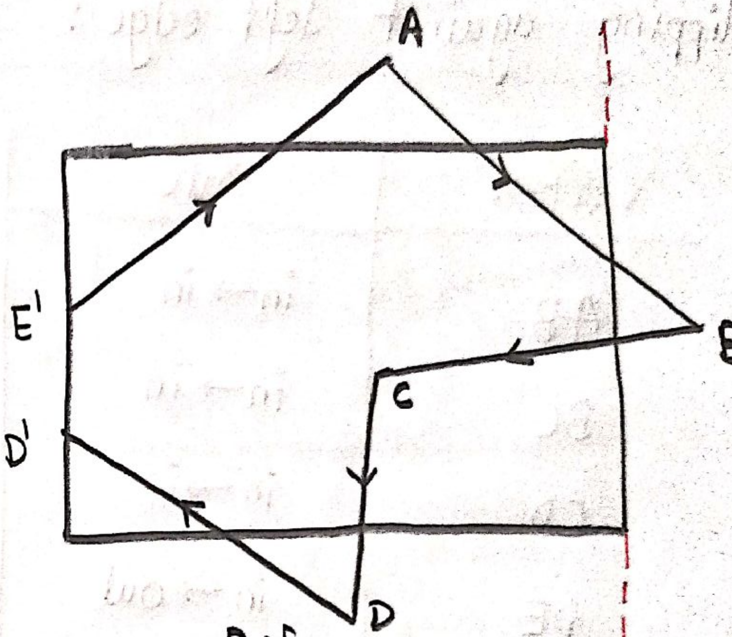
Before



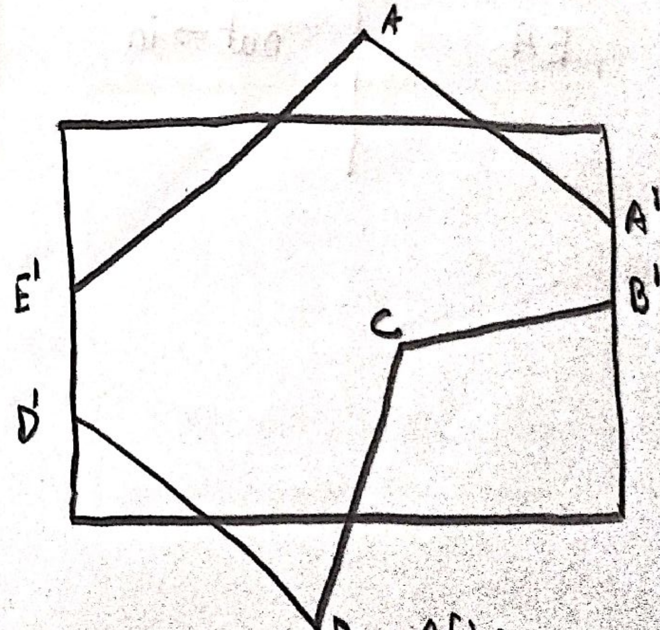
After

② Clipping against right edge :

Vertex	Rule	Output
AB	in→out	A'
BC	out→in	B'C
CD	in→in	D
DD'	in→in	D'
D'E'	in→in	E'
E'A	in→in	A



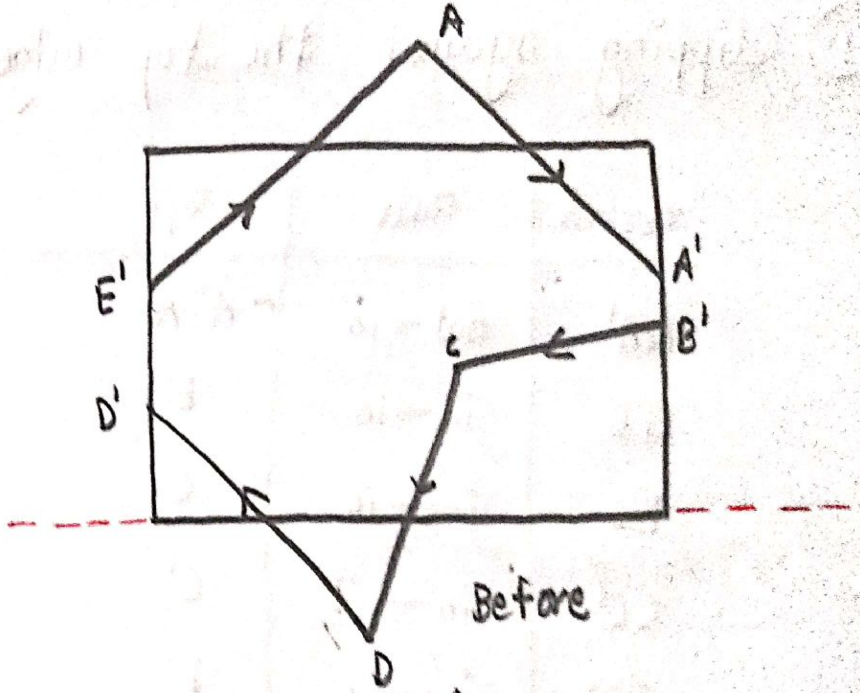
Before



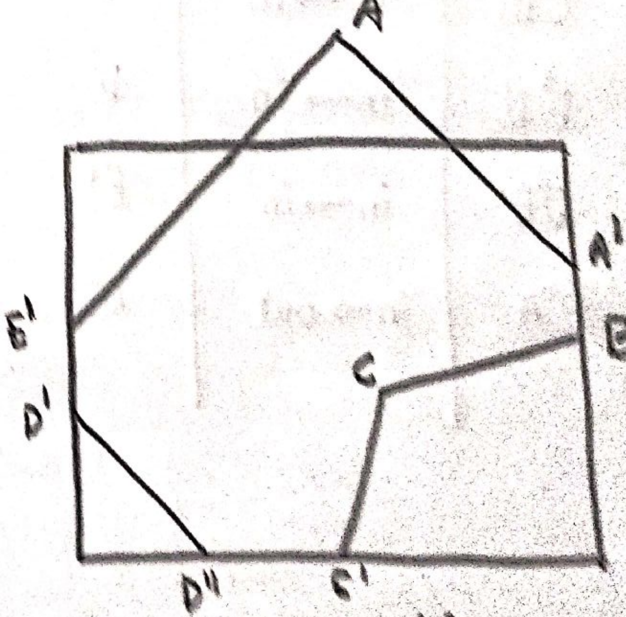
After

③ Clipping against bottom edge:

vertex	Rule	Output
AA'	in→in	A'
A'B'	in→in	B'
B'C	in→in	C
CD	in→out	C'
DD'	out→in	D''D'
D'E	in→in	E'
E'A	in→in	A



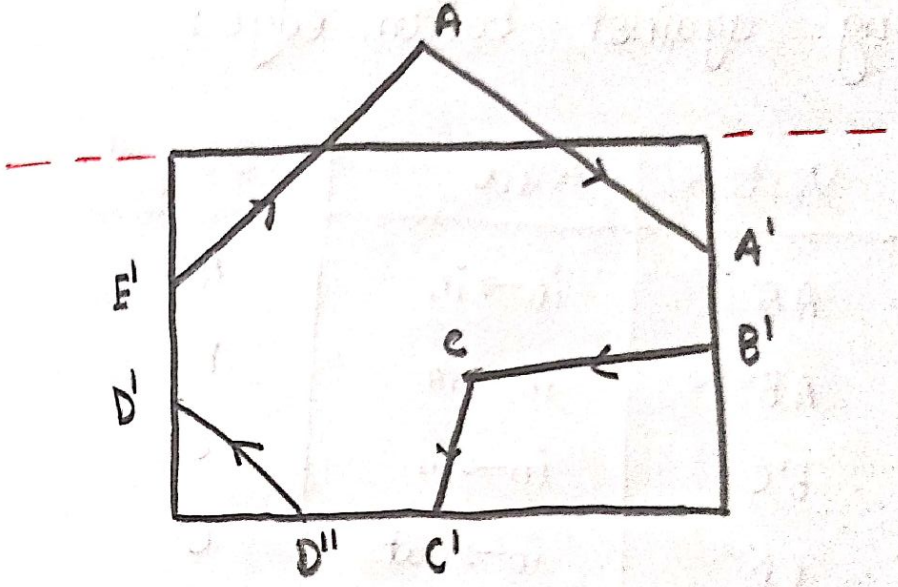
Before



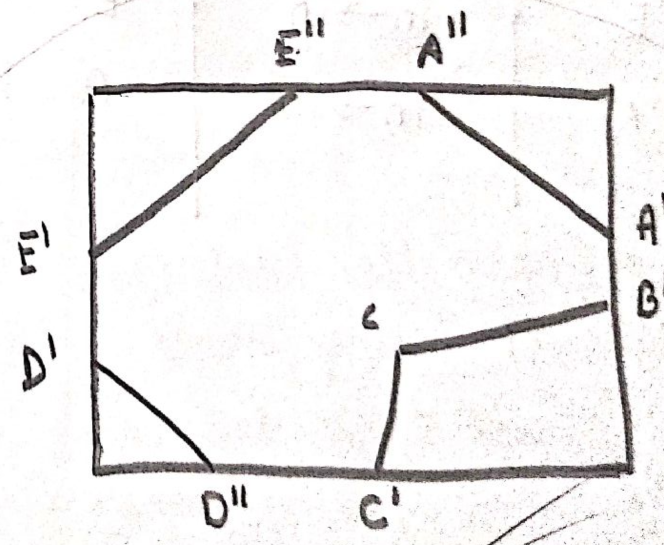
After

④ Clipping against the top edge :

Vertex	Rule	O/P
AA'	out → in	A" A'
A'B'	in → in	B'
B'C	in → in	C
CC'	in → in	C'
C'D"	in → in	D"
D"D'	in → in	D'
D'E'	in → in	E'
E'A	in → out	E"



Before



Algorithm :

```
① Input four clippers & $C_l = X_{min}$,
   $C_r = X_{max}$, $C_t = Y_{max}$, $C_b = Y_{min}$
   corresponding to the left, right,
   top, and bottom window boundaries.
   The polygon is specified in term of
   its vertex list $V_{in} = \{V_1, V_2, \dots V_n\}$
   where the vertices are named anticlockwise.

② For each clipper in the order $C_l, C_r, C_t, C_b$ do
③ Set output vertex list $V_{out} = NULL$, $i=1$, $j=1$
④ repeat
⑤ Consider the vertex pair $v_i$ & $v_j$ in $V_{in}$
⑥ if $v_i$ is inside and $v_j$ outside then
      ADD the intersection point of the
      clipper with the edge $(v_i, v_j)$ to $V_{out}$
⑦ else if both the vertices are
   inside the clipper then
⑧ ADD $v_j$ to $V_{out}$
⑨ else if $v_i$ is outside & $v_j$
   inside of the clipper then
⑩ ADD the intersection point of
   the clipper with the edge $(v_i, v_j)$
   and $v_j$ to $V_{out}$
⑪ else
      ADD NULL to $V_{out}$
⑫ end if
⑬ Until all edge (ie consecutive
   vertex pairs) in $V_{in}$ are checked.
⑭ Set $V_{in} = V_{out}$
⑮ End for.
⑯ Return $V_{out}$.
```

Steps of Sutherland Hodgeman Polygon Clipping Algorithm:

① Read the coordinates of all vertices of the polygon.

② Read the coordinates of ~~at~~ the clipping window.

③ Consider the left edge of the window.

④ Compare the vertices of the edge of the polygon, individually with the clipping plane.

⑤ Save the resulting intersections and the vertices according to the rule.

⑥ Repeat step ④, ⑤ for the remaining edges of the clipping window.

⑦ Each time pass the resultant list of vertices to the next edge of the clipping window.

⑧ STOP.